Physics with neutrons 2

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Exercise sheet 6
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EXERCISE 6.1

Calculate $\langle u^2 \rangle_T$ and f^2_{DWF} for lead ($\theta_D = 88\,\text{K}$), copper ($\theta_D = 315\,\text{K}$), and diamond ($\theta_D = 1860\,\text{K}$) at $T = 10\,\text{K}$ and $T = 1000\,\text{K}$ with the low and high temperature approximations. Which material is most useful as a monochromator? What can be done to improve the reflectivity of copper monochromators?

EXERCISE 6.2

Prove that spin-incoherent scattering is $\frac{2}{3}$ spin-flip scattering and $\frac{1}{3}$ non-flip scattering. This can be done by the following steps:

1. Start with the expression given in the lecture for $\overline{b}$ in (A.2.10), assuming a single isotope with nuclear spin I. Express b^+ and b^- in the form

$$b^+ = \overline{b} + g^+(I) \cdot B \qquad \text{and}$$
$$b^- = \overline{b} + g^-(I) \cdot B \qquad \text{with} \quad B = \frac{b^+ - b^-}{2I+1},$$

   i. e. find $g^+(I)$ and $g^-(I)$.

2. Now we want to unify $g^+(I)$ and $g^-(I)$ so that we can give a single expression for b. Do so using the projection operator P that projects the spin of the neutron from its initial quantization axis onto a new quantization axis in direction of $\vec{I} = (I_x, I_y, I_z)$, which is

$$P(\vec{I}) = 1 + I_x\sigma_x + I_y\sigma_y + I_z\sigma_z = 1 + \vec{I} \cdot \sigma.$$

   The new axis $\vec{I}$ is the quantization axis of the nucleus (neutron and nucleus have to have the same quantization axis to decide if they are parallel or antiparallel). $\sigma_{x,y,z}$ are the Pauli matrices and σ is a vector of the Pauli matrices.

3. This yields an expression for the scattering length of a nucleus using the mean value of parallel and antiparallel neutron-nucleus spin alignment and the deviation thereof. Which part of this scattering length will give rise to incoherent scattering? Write the incoherent cross section.

4. We are now not only interested in the probability that there is *some* incoherent scattering event but want to split it up in probabilities for scattering events where the neutron has a spin when incoming $|s_i\rangle$ and when outgoing $|s_f\rangle$. These spin states are prepared / measured in polarizers / detectors that are sensitive in the laboratory z-direction. Therefore the two spin states can both be either up $\binom{1}{0}$ or down $\binom{0}{1}$. The probabilities (cross sections) can be calculated using

$$\sigma_{|s_i\rangle\to|s_f\rangle} = 4\pi \left|b_{|s_i\rangle\to|s_f\rangle}\right|^2 \propto \left|\langle s_f|\vec{I}\sigma|s_i\rangle\right|^2 = \left|\langle s_f|I_x\sigma_x|s_i\rangle + \langle s_f|I_y\sigma_y|s_i\rangle + \langle s_f|I_z\sigma_z|s_i\rangle\right|^2.$$

   Calculate $b_{|\uparrow\rangle\to|\uparrow\rangle}$, $b_{|\uparrow\rangle\to|\downarrow\rangle}$,$b_{|\downarrow\rangle\to|\uparrow\rangle}$, and $b_{|\downarrow\rangle\to|\downarrow\rangle}$ as functions of the nuclear spin components I_x, I_y, and I_z.

5. Assume that the nuclei are not aligned, therefore $I_x^2 = I_y^2 = I_z^2$ to calculate the relative frequencies of spin flip and non spin flip scattering.

Exercise 6.3

Derive the representation

$$G(\mathbf{r},t) = \frac{1}{N}\sum_{j,j'}\int \langle\delta\Big(\mathbf{R}-\mathbf{r}_{j'}(0)\Big)\delta\Big(\mathbf{R}+\mathbf{r}-\mathbf{r}_j(t)\Big)\rangle dR$$

from the expression for the intermediate scattering function

$$I(\mathbf{Q},t) = \frac{1}{N}\sum_{j,j'}\langle e^{-i\mathbf{Q}\cdot\mathbf{r}_{j'}(0)}e^{i\mathbf{Q}\cdot\mathbf{r}_j(t)}\rangle_T$$

using the substitution

$$e^{-i\mathbf{Q}\cdot\mathbf{r}_{j'}(0)} = \int e^{-i\mathbf{Q}\cdot\mathbf{r}'}\delta\Big(\mathbf{r}'-\mathbf{r}_{j'}(0)\Big)d\mathbf{r}'.$$