Physics with neutrons 1

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Winter semester 2015/16
Exercise sheet 4
Due 2015–Nov–13

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Exercise 4.1

Calculate the magnetic interaction potential for a typical rare earth magnet, $\mu \approx 10\,\mu_B$ (Tb^{3+}, Dy^{3+}, Ho^{3+}), and of a Cu spin in a high-T_c superconductor, $\mu \approx 1\,\mu_B$, in a field of 1 T.

Exercise 4.2

The potential

$$U(r, \vartheta, \varphi) = -U_0\,\Theta(R - r)$$

is called a hard sphere potential with radius R. ($\Theta(x)$ is the Heaviside step function, which is defined to be zero for $x < 0$ and unity for $x \geq 0$.)

1. Calculate the differential and the total cross section of scattering from this potential.
2. Using small-angle neutron scattering, a biologist would like to measure the diameter of spherical micelles (aggregated "clusters" of molecules in a solvent). What is the form factor $F(QR)$ (i.e. the Q-dependent part of the differential scattering cross section) of one such micelle under the assumption that it can be approximated by a homogeneous sphere with a radius of 200 nm?
3. For small values of QR, the form factor can be Taylor-expanded. What is the resulting behavior?
4. Plot the form factor (versus QR) on a log-log scale. For large values of QR, what is the behavior of $F(QR)$ when one averages over the oscillations?
5. What happens (qualitatively) when the sphere is placed in a solvent? What happens when there are multiple spheres present?

Exercise 4.3

Electrons are the origin of the magnetism in magnetic materials. Assume that the probability density of finding an electron at $\vec{r} = (r, \vartheta, \varphi)$ is given by a Gaussian profile

$$\rho(r) = \rho_0 \exp\left[-\frac{r^2}{2\sigma^2}\right]$$

with a half-width at half maximum of $a = 2\,\text{Å}$ (what is the relation between σ and a?). From this profile, calculate the magnetic form factor of an unpaired electron.