Physics with neutrons 2

Sebastian Mühlbauer, sebastian.muehlbauer@frm2.tum.de
Sommer semester 2016
Exercise sheet 1
Due 2016–Apr–22

Lukas Karge, lukas.karge@frm2.tum.de, Tel.: 089-289-11774

Exercise 1.1

Estimate the energy scale of the magnetic interaction for

- two electrons,
- an electron and a neutron,
- an electron and a nucleus (for example Cu), and
- a neutron and a nucleus (for example In).

The respective particles are supposed to have a distance of 1 Å.

Solution. The magnetic interaction energy is given by

$$\hat{U}_m(\mathbf{R}) = -\hat{\mu} \cdot \mathbf{B}(\mathbf{R}),$$

where $\hat{\mu}$ is the magnetic moment operator. We use the expression for the field $\mathbf{B}(\mathbf{R})$ of an electron for a spin-only system and get:

$$U_m(\mathbf{R}) = -\mu_1 \cdot \frac{\mu_0}{4\pi}\left(\nabla \times \frac{\mu_2 \times \mathbf{R}}{R^3}\right) = -\mu_1 \cdot \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{R}(\mu_2 \cdot \mathbf{R})}{R^5} - \frac{\mu_2}{R^3}\right).$$

Assuming both moments to be collinear, and perpendicular to $\mathbf{R}$, we get

$$U_m(R) = \frac{\mu_0}{4\pi}\frac{\mu_1\mu_2}{R^3}.$$

The magnetic moments of the given particles and atoms are:

Particle	Magnetic moment
Neutron	$-1.91\,\mu_N$
Electron	$-1.001\,\mu_B$
^{63}Cu	$2.22\,\mu_N$
^{65}Cu	$2.38\,\mu_N$
^{113}In	$5.53\,\mu_N$

(μ_B is the Bohr magneton, $e\hbar/2m_e = 9.274 \times 10^{-24}$ J/T. μ_N is the nuclear magneton, $e\hbar/2m_p = 5.051 \times 10^{-27}$ J/T. The nuclear moments of isotopes can readily be found on the web, e.g. on webelements.com.)

Now we can calculate the magnetic interaction of the given systems:

System	μ_1 (10^{-27} J/T)	μ_2 (10^{-27} J/T)	U_m (µeV)
e-e	-9285	-9285	53.8
e-n	-9285	-9.66	0.056
e-^{63}Cu	-9285	12.3	-0.065
e-^{65}Cu	-9285	12.8	-0.070
n-^{113}In	-9.66	27.9	-1.68×10^{-4}

As one can see, the interaction between neutrons and the nuclear spins is about 1000 times weaker than between neutrons and free electrons. It still plays a role in spin-incoherent scattering though. □

Exercise 1.2

1. With which part of matter interact neutrons and photons, respectively? What are the differences between light and neutron scattering?
2. What gives rise to coherent / incoherent scattering? Which information can be extracted from each of them?
3. Recall the most important facts about the structure factor $|S|^2$ and the form factor $|F|^2$.

Solution.

1. Whereas photons are scattered by the electrons, neutrons interact with the nuclei (and magnetic moments). Due to the mass of the neutron, the accessible momentum transfers are very different in an experiment (important for phonons).
2. The scattering of one scatterer is per se neither coherent nor incoherent. The part with a modulated intensity reflecting the internal structure of the sample is termed coherent. If the scatterers in the sample have different scattering cross sections (which is especially for neutrons the normal case as the cross section depends on the relative alignment of the spins), a part of the scattering does not completely destructively interfere so that it yields a Q independent term, the incoherent scattering.

   In elastic scattering, the incoherent term does not carry any information – this changes in inelastic scattering where the coherent term yields information about the pair correlation function and the incoherent term about the auto correlation function.
3. Both have in common that only the square of the absolute value can be measured.

   The structure factor $|S|^2$ describes the alignment of the unit cells.
   - It peaks at each reciprocal lattice vector. The corresponding peaks are called *Bragg peaks.*
   - Their height is $\propto N^2$, their width $\propto \frac{1}{N_k}$.
   - From the peak positions, one can determine
     - crystal system
     - Bravais lattice (from systematic extinctions)
     - macro-strain
     - lattice constants.
   - From the peak width, one obtains information about
     - particle size
     - shape of the particle
     - internal micro-strain.

   The form factor $|F|^2$ describes the interior of the unit cell.
   - It can only be measured as intensity of the reflections at the reciprocal lattice vectors (where the structure factor is not zero).
   - It is the squared Fourier transform of the scattering potential.
   - If the scattering results from a non-point-like particle (e. g. electrons for photons or magnetic moments for neutrons), the form factor diminishes with increasing Q, rendering only a few peaks at low Q observable (Fourier transform of unlike δ-function).
   - The fact that the phases cannot be measured is called the *phase problem.* There are several methods to tackle it but it is never possible to completely resolve them.

□