Physics with neutrons 2

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Sommer semester 2016
Exercise sheet 2
Due 2016–Apr–29

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Exercise 2.1

When investigating an object with small-angle scattering, the Patterson function yields useful statistical information. It is defined by

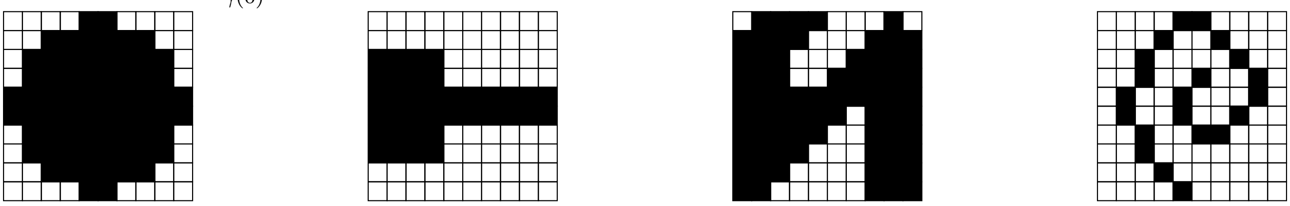
$$P(\mathbf{r}) = \int \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\, d\mathbf{V}, \qquad \text{with } \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,$$

where ρ is the scattering-length density function of the object. The correlation function $\gamma(r)$ is the orientational average of the Patterson function $P(\mathbf{r})$, which is in two dimensions

$$\gamma(r) = \frac{1}{2\pi}\int_0^{2\pi} P(\mathbf{r})\,\mathrm{d}\varphi \ . \tag{1}$$

It answers the question: given that there is an atom of the particle at some place, what is the probability that the atoms in the distance r are also situated inside the particle?

Numerically calculate the (two-dimensional) Patterson function and subsequently the characteristic function $\gamma_0(r) = \frac{\gamma(r)}{\gamma(0)}$ of the following objects (black area $\rho = 1$, white area $\rho = 0$)



Why is $\gamma(r \geq D) = 0$ when D is the largest possible distance of two atoms inside the particle? What is the connection between Patterson function and scattering signal of the object?

Exercise 2.2

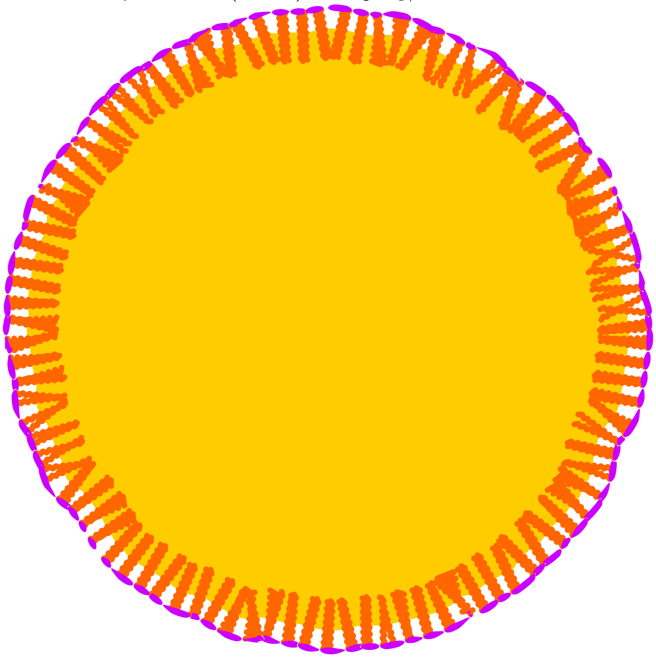
To obtain *Porod's law*, an intermediate step is to integrate the central formula for the scattered intensity I as a function of the absolute value of the scattering vector Q,

$$I(Q) = 4\pi \int_0^D \gamma(r)\frac{\sin(Qr)}{Q} r\,\mathrm{d}r \ . \tag{2}$$

Solve the integral.

Exercise 2.3

Consider an emulsion of hexane droplets (diameter about 100 nm) in water, stabilized by the (protonated) phospholipid DMPC. Hexane (C_6H_{14}) has a density of 659 kg/m³, Hexane-d14 (C_6D_{14}) 767 kg/m³, water (H_2O) 1000 kg/m³, and heavy water (D_2O) 1105 kg/m³.



The aim is to obtain as much information as possible about the structure of the emulsion using small angle neutron scattering.

1. Which typical distances can be found in the sample and which of them should be observable by SANS? Assume in the following that the concentration of the emulsion droplets is very small.

2. What are the contributions of coherent and incoherent scattering, respectively?

3. In which range can you vary the scattering length density (SLD) of water and hexane by mixing protonated and deuterated substances?

4. How would you choose the scattering length densities if you would like to determine the thickness of the stabilizer layer?