Physics with neutrons 2

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Exercise sheet 9
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Exercise 9.1

Prove that spin-incoherent scattering is $\frac{2}{3}$ spin-flip scattering and $\frac{1}{3}$ non-flip scattering. This can be done by the following steps:

1. Start with the expression given in the lecture and derived in exercise 6.1 for $\bar{b}$, assuming a single isotope with nuclear spin I. Express b^+ and b^- in the form

$$b^+ = \bar{b} + g^+(I) \cdot B \qquad \text{and}$$
$$b^- = \bar{b} + g^-(I) \cdot B \qquad \text{with} \quad B = \frac{b^+ - b^-}{2I+1},$$

   i. e. find $g^+(I)$ and $g^-(I)$.

2. Now we want to unify $g^+(I)$ and $g^-(I)$ so that we can give a single expression for b. Do so using the projection operator P that projects the spin of the neutron from its initial quantization axis onto a new quantization axis in direction of $\vec{I} = (I_x, I_y, I_z)$, which is

$$P(\vec{I}) = 1 + I_x\sigma_x + I_y\sigma_y + I_z\sigma_z = 1 + \vec{I} \cdot \sigma.$$

   The new axis $\vec{I}$ is the quantization axis of the nucleus (neutron and nucleus have to have the same quantization axis to decide if they are parallel or antiparallel). $\sigma_{x,y,z}$ are the Pauli matrices and σ is a vector of the Pauli matrices.

3. This yields an expression for the scattering length of a nucleus using the mean value of parallel and antiparallel neutron-nucleus spin alignment and the deviation thereof. Which part of this scattering length will give rise to incoherent scattering? Write the incoherent cross section.

4. We are now not only interested in the probability that there is *some* incoherent scattering event but want to split it up in probabilities for scattering events where the neutron has a spin when incoming $|s_i\rangle$ and when outgoing $|s_f\rangle$. These spin states are prepared / measured in polarizers / detectors that are sensitive in the laboratory z-direction. Therefore the two spin states can both be either up $\binom{1}{0}$ or down $\binom{0}{1}$. The probabilities (cross sections) can be calculated using

$$\sigma_{|s_i\rangle\to|s_f\rangle} \;=\; 4\pi \left|b_{|s_i\rangle\to|s_f\rangle}\right|^2 \;\propto\; \left|\langle s_f|\vec{I}\sigma|s_i\rangle\right|^2 \;=\; \left|\langle s_f|I_x\sigma_x|s_i\rangle \;+\; \langle s_f|I_y\sigma_y|s_i\rangle \;+\; \langle s_f|I_z\sigma_z|s_i\rangle\right|^2.$$

   Calculate $b_{|\uparrow\rangle\to|\uparrow\rangle}$, $b_{|\uparrow\rangle\to|\downarrow\rangle}$,$b_{|\downarrow\rangle\to|\uparrow\rangle}$, and $b_{|\downarrow\rangle\to|\downarrow\rangle}$ as functions of the nuclear spin components I_x, I_y, and I_z.

5. Assume that the nuclei are not aligned, therefore $I_x^2 = I_y^2 = I_z^2$ to calculate the relative frequencies of spin flip and non spin flip scattering.

Exercise 9.2

The quaternary intermetallic compound $HoNi_2B_2C$ exhibits a coexistence of superconductivity and long-range magnetic ordering of the Ho^{3+} sublattice at low temperatures. The compound has a body-centered tetragonal structure as shown in Fig. 1a. The structural parameters in the paramagnetic state at $T = 10$ K are $a = 3.5087$ Å and $c = 10.5274$ Å, with atomic positions Ho $(0, 0, 0)$, Ni $(1/2, 0, 1/4)$, B $(0, 0, z = 0.3592)$ and C $(1/2, 1/2, 0)$. The transition into the superconducting state occurs below $T_c \approx 8$ K. At the same temperature the compound transforms from the paramagnetic state into a magnetically ordered state. The magnetic structure is a superposition of two configurations, one in which the Ho^{3+} spins are coupled antiferromagnetically (Fig. 1b) and another which corresponds to a helical spin structure (Fig. 1c). The relative contribution of the helical spin structure is strongly reduced with decreasing temperature and at $T = 2.2$ K only the antiferromagnetic structure is present as shown in Fig. 1b. The helical spin structure manifests itself by pairs of magnetic satellites around the magnetic Bragg peaks as visualized in Fig. 2(r) for the $(0, 0, 1)$ reflection observed at $T = 5.1$ K.

1. Determine the antiferromagnetic structure of $HoNi_2B_2C$ from the neutron diffraction pattern observed at $T = 2.2$ K (Fig. 2(l)).

2. Determine the magnetic ordering wavevector $\mathbf{q}_0$ associated with the magnetic spiral structure of $HoNi_2B_2C$ from the neutron diffraction pattern observed at $T = 5.1$ K (Fig. 2(r)). What is the turn angle ϕ of the spins between adjacent planes?

Exercise 9.3

Prove that a Dzyaloshinskii-Moriya helimagnet can be treated as an easy-plane antiferromagnet for a "right" angle ϕ between z component of spin and the Dzyaloshinskii-Moriya vector.

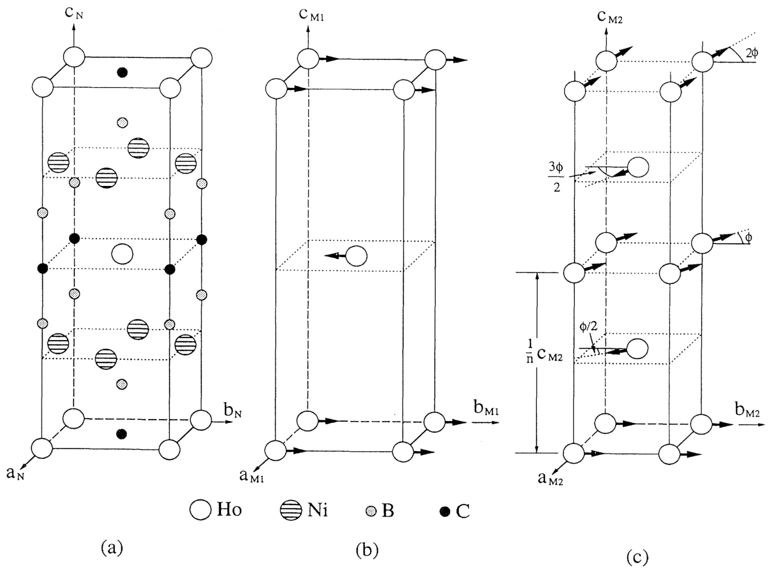

Figure 1: Schematic representation of (a) the unit cell of $HoNi_2B_2C$; (b) the antiferromagnetic ordering of the Ho^{3+} spins; (c) the spin configuration of the magnetic helical structure. [Huang et al. (1995)]

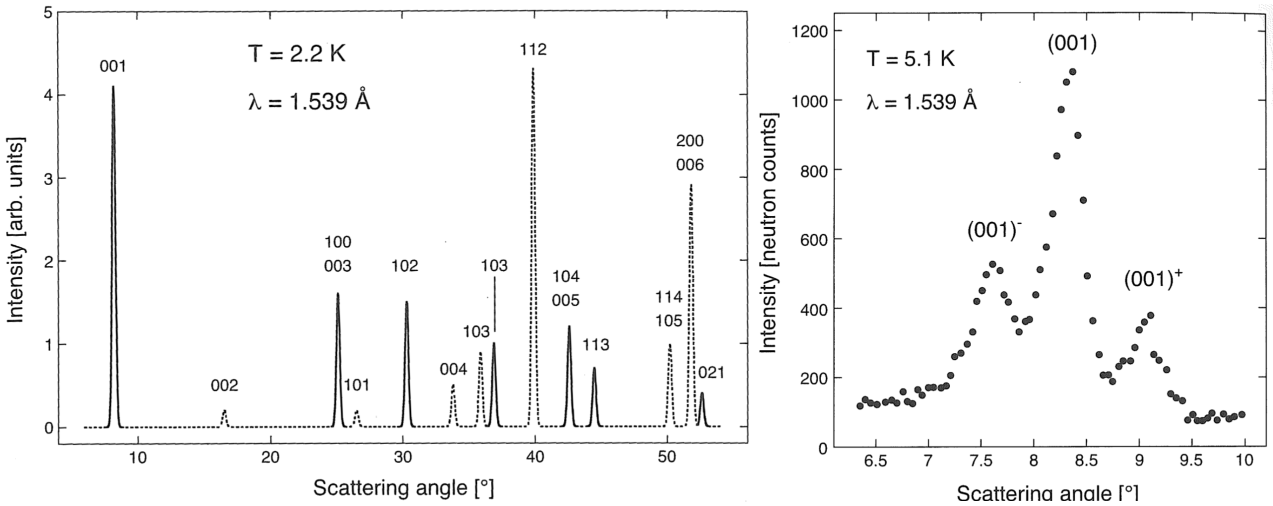

Figure 2: (l) Powder diffraction pattern of $HoNi_2B_2C$. Dotted lines nuclear, solid lines magnetic peaks. (r) Low-angle part of the diffraction pattern of $HoNi_2B_2C$.