Physics with neutrons 1

Michael Leitner, michael.leitner@frm2.tum.de
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Exercise sheet 2
To be discussed 2016-11-04, room C.3202

Franz Haslbeck, franz.haslbeck@frm2.tum.de

Exercise 2.1

Calculate the binding energies of the isotopes from hydrogen up to carbon-14 in MeV using the mass defect. Draw the binding energy per nucleon as function of mass number A. Which atoms are stable considering the binding energies and beta decay? Why has the isotope Be-8 an extremely short half-life?

Solution. The binding energy can be calculated using

$$B[MeV] = [Z \cdot (m_p + m_e) + N \cdot m_n - m_{isotope}] \cdot 931.49 \, \frac{\text{MeV}}{\text{u}} \tag{1}$$

with $m_p = 1.007276\,\text{u}$, $m_n = 1.008665\,\text{u}$ and $m_e = 0.000549\,\text{u}$.
Tritium decays with a half-life of 12,32 a into He-3, an electron and an electron-antineutrino. In this beta-decay 18.6 keV of energy are released. Be-8 decays via alpha-decay into two alpha particles releasing 92 keV.

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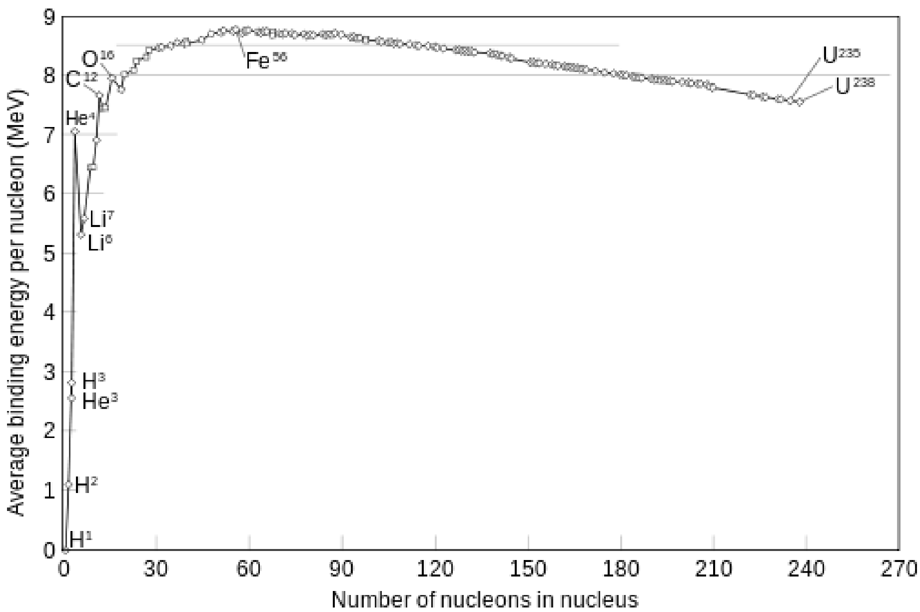

Figure 1: Binding energy per nucleon B/A as function of mass number A. Graph taken from `https://de.wikipedia.org/wiki/Bindungsenergie`

Exercise 2.2

Nuclei with non-zero nuclear spin contribute to spin incoherence as the neutron and nuclear spin can be either parallel or antiparallel. Calculate the weighting factors A.2.10 from the lecture and the scattering length b_+ for parallel spin alignment as well as b_- for antiparallel spin alignment of hydrogen. Coherent and incoherent scattering lengths for any element can be found at `http://www.ncnr.nist.gov/resources/n-lengths/`.

Solution. The neutron is a fermion with spin 1/2 and couples to the nuclear spin $I = 1/2$ of the hydrogen nucleus. Therefore, the system can be in a triplet state $\vec{I} + \frac{\vec{1}}{2}$ or a singlet state $\vec{I} - \frac{\vec{1}}{2}$. The number of degenerate states is

$$2(I + \frac{1}{2}) + 1 = 2I + 2 \tag{2}$$

and

$$2(I - \frac{1}{2}) + 1 = 2I \tag{3}$$

which leads to $2(2I + 1)$ degenerate states. Hence, the weighting factors as given in the lecture are

$$w_+ = \frac{2I+2}{4I+2} = \frac{I+1}{2I+1} \qquad \text{and} \qquad w_- = \frac{2I}{4I+2} = \frac{I}{2I+1}. \tag{4}$$

The averages over spin states are calculated for the coherent and incoherent scattering using

$$\langle b \rangle = w_+ b_+ + w_- b_- = \frac{(I+1)b_+ + I b_-}{2I+1} \tag{5}$$

$$\langle b^2 \rangle = w_+ b_+^2 + w_- b_-^2 = \frac{(I+1)b_+^2 + I b_-^2}{2I+1}. \tag{6}$$

The coherent and incoherent scattering lengths are given as

$$b_{coh} = \langle b \rangle = w_+ b_+ + w_- b_- \tag{7}$$

$$b_{inc} = \sqrt{\langle b^2 \rangle - \langle b \rangle^2} = \sqrt{w_+ w_-}(b_+ - b_-). \tag{8}$$

□

Rearranging the formulas

$$b_+ = b_{coh} + \sqrt{\frac{w_-}{w_+}} b_i = b_{coh} + \sqrt{\frac{I}{I+1}} b_{inc} \tag{9}$$

$$b_- = b_{coh} + \sqrt{\frac{w_+}{w_-}} b_i = b_{coh} - \sqrt{\frac{I+1}{I}} b_{inc} \tag{10}$$

and inserting $b_{coh} = -3.7406\,\text{fm}$, $b_{inc} = 25.274\,\text{fm}$, hence gives $b_+ = 10.851\,\text{fm}$ and $b_- = -47.517\,\text{fm}$