Physics with neutrons 2

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Summer semester 2017
Exercise sheet 3
To be discussed 2017-05-16, room C.3203

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EXERCISE 3.1

The potential

$$U(r,\vartheta,\varphi) = -U_0\,\Theta(R-r)$$

is called a hard sphere potential with radius R. ($\Theta(x)$ is the Heaviside step function, which is defined to be zero for $x < 0$ and unity for $x \geq 0$.)

1. Calculate the differential and the total cross section of scattering from this potential.
2. Using small-angle neutron scattering, a biologist would like to measure the diameter of spherical micelles (aggregated "clusters" of molecules in a solvent). What is the form factor $F(QR)$ (i.e. the Q-dependent part of the differential scattering cross section) of one such micelle under the assumption that it can be approximated by a homogeneous sphere with a radius of 200 nm?
3. For small values of QR, the form factor can be Taylor-expanded. What is the resulting behavior?
4. Plot the form factor (versus QR) on a log-log scale. For large values of QR, what is the behavior of $F(QR)$ when one averages over the oscillations?
5. What happens (qualitatively) when the sphere is placed in a solvent? What happens when there are multiple spheres present?

EXERCISE 3.2

When investigating an object with small-angle scattering, the Patterson function yields useful statistical information. It is defined by

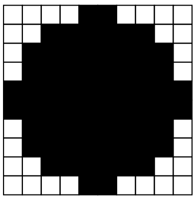
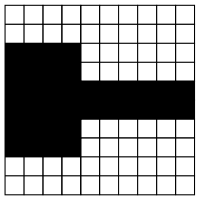
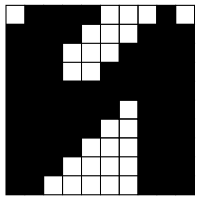
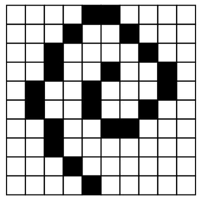
$$P(\mathbf{r}) = \int \rho(\mathbf{r}_1)\rho(\mathbf{r}_2)\,d\mathbf{V}, \qquad \text{with } \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2,$$

where ρ is the scattering-length density function of the object. The correlation function $\gamma(r)$ is the orientational average of the Patterson function $P(\mathbf{r})$, which is in two dimensions

$$\gamma(r) = \frac{1}{2\pi}\int_0^{2\pi} P(\mathbf{r})\,\mathrm{d}\varphi\,. \tag{1}$$

It answers the question: given that there is an atom of the particle at some place, what is the probability that the atoms in the distance r are also situated inside the particle?

Numerically calculate the (two-dimensional) Patterson function and subsequently the characteristic function $\gamma_0(r) = \frac{\gamma(r)}{\gamma(0)}$ of the following objects (black area $\rho = 1$, white area $\rho = 0$)

Why is $\gamma(r \geq D) = 0$ when D is the largest possible distance of two atoms inside the particle? What is the connection between Patterson function and scattering signal of the object?