Physics with Neutrons I

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Exercise sheet 11 https://wiki.mlz-garching.de/n-lecture06:index

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1. Specular reflectivity

Assume a single planar interface between vacuum and a material with refractive index n. Derive the reflectivity of a neutron beam in the case of specular reflection and compute it for neutrons with wavelength $\lambda = 4\,$Åand $n = 1 \pm 10^{-5}$. All neutron beams can be assumed as planar waves and the boundary conditions at the interface are the continuity of the wave function and its derivative. Use, that the wave vector changes due to the refractive index and that all components parallel to the surface stay constant at the interface.
What happens for $n < 1$ mathematically and what is the physical interpretation?

The reflectivity can also be derived starting from a scattering point of view. The crucial steps are to start from the differential scattering cross section and evaluate this for $Q_x = Q_y = 0$ and a scattering length density which only depends on z. Still, this has to be integrated over a small solid angle which represents the inherent uncertainty of Q. The result is the following approximation:

$$R(Q) \approx \frac{16\pi^2}{Q^2}\Big|\int_{-\inf}^{\inf} \Delta\rho(z) e^{-i\,Q_z\,z} dz\Big|^2$$

This formula has a problem for 'infinitely thick' materials, because the integral will not converge. This is caused by the unphysical assumption that we neglect absorption. The problem can be solved by simply dropping the integration terms evaluated at $z = \pm\inf$.
Use this approximation to calculate the reflectivity of the same interfaces as given above. Where is the approximation valid and which important effect is not taken into account?

Compared to the optical approach, it is easier to calculate the effect of multiple interfaces with the scattering approximation. How does the reflectivity change, if the sample has finite thickness (e.g. $10\,\mu$m)? What is the physical interpretation?