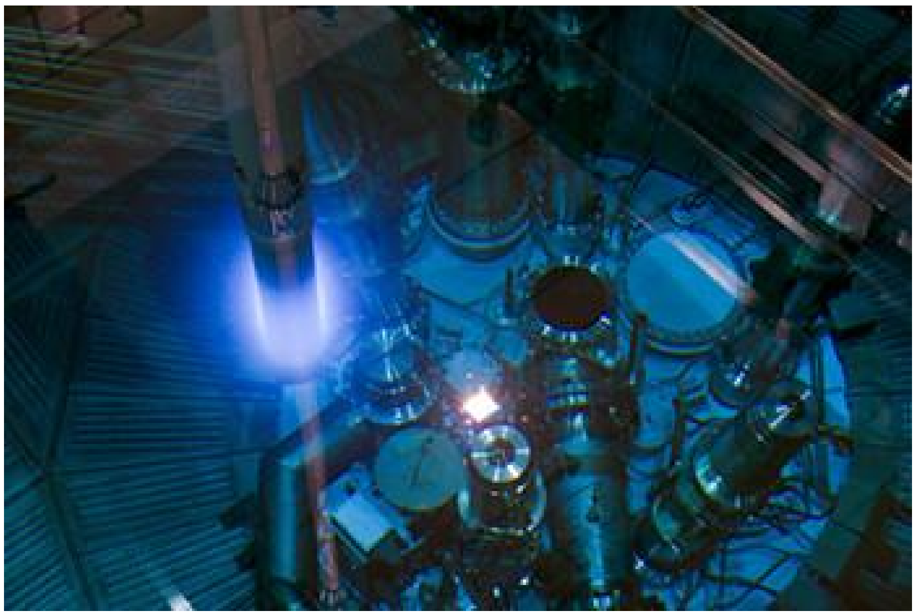
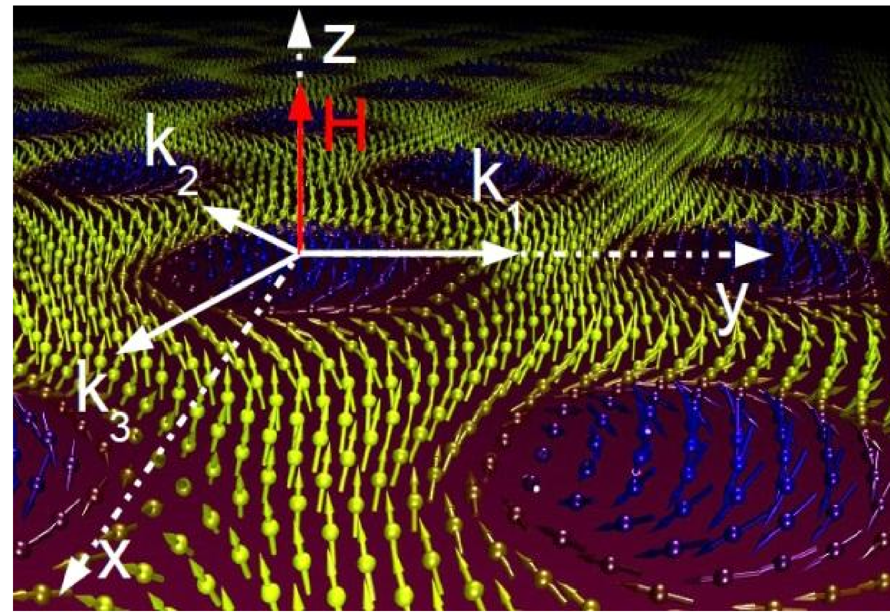
FRM II
Forschungs-Neutronenquelle
Heinz Maier-Leibnitz

MLZ
Heinz Maier-Leibnitz Zentrum

# Physics with Neutrons I, WS 2015/2016






## Lecture 5, 16.11.2015

MLZ is a cooperation between:

JÜLICH FORSCHUNGSZENTRUM

Helmholtz-Zentrum Geesthacht
Zentrum für Material- und Küstenforschung

TUM
Technische Universität München

# Organization



## Visit FRM II : 21.12.2015

10:15 – 13:00 (after the lecture)

Valid ID necessary!!!

## Exam (after winter term)

- Registration: via TUM-Online between 16.11.2015 – 15.1.2015
- Email: sebastian.muehlbauer@frm2.tum.de for date arrangement
- 30min oral exam

## 3.2 Fermis Golden Rule - reminder

Fermis Golden Rule for scattering process: Born approximation

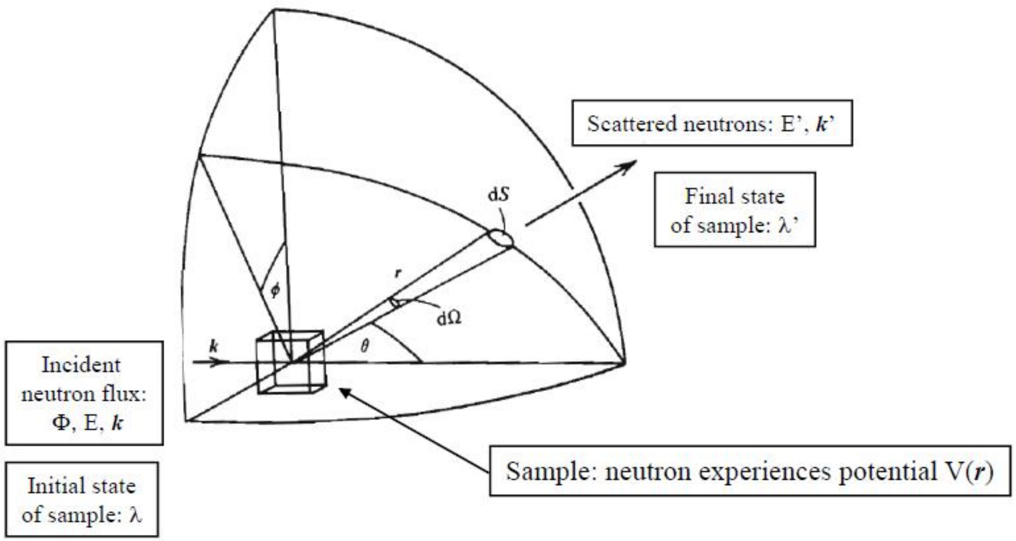
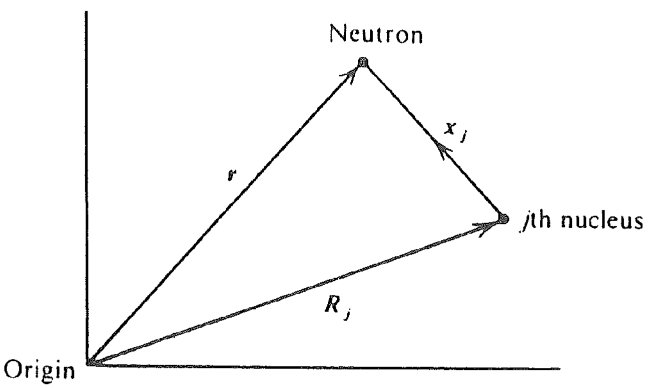



Fig. 2.1 Coordinates of nucleus and neutron.




$$\left(\frac{d\sigma}{d\Omega}\right)_{\lambda\to\lambda'} = \frac{1}{\Phi}\frac{1}{\Omega}\sum_{k'\ in\ d\Omega} W_{k,\lambda\to k',\lambda'}$$

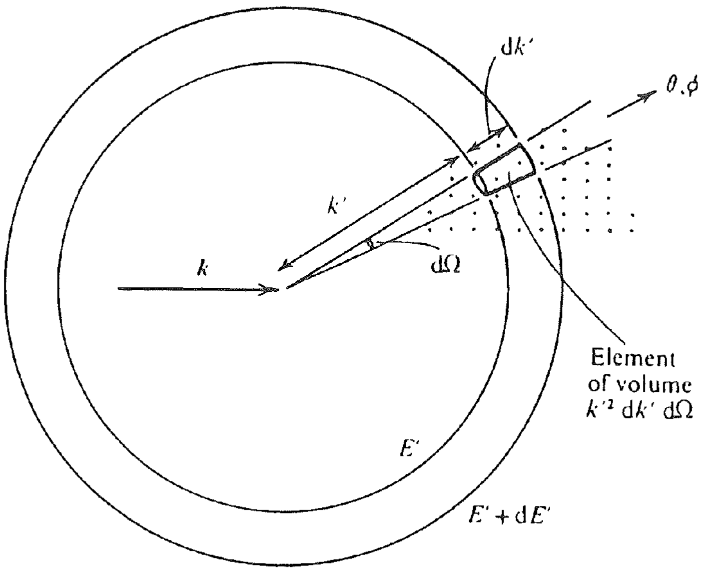
$$\sum_{k'\ in\ d\Omega} W_{k,\lambda\to k',\lambda'} = \frac{2\pi}{\hbar}\varrho_k \left|\langle k'\lambda'|V|k\lambda\rangle\right|^2$$

$$\langle k'\lambda'|V|k\lambda\rangle = \int \Psi^*_{k'}\chi^*_{\lambda'} V \Psi^*_k \chi^*_\lambda \mathrm{d}r\mathrm{d}R$$

Challenge: Rewrite this mixed expression in terms of sample properties

## 3.2 Fermis Golden Rule - reminder

1st step: Box Normalization to calculate $\rho_k$




$$\left(\frac{d\sigma}{d\Omega}\right)_{\lambda\to\lambda'} = \frac{k'}{k}\left(\frac{m}{2\pi\hbar}\right)^2 |\langle k'\lambda'|V|k\lambda\rangle|^2$$

## 3.2 Fermis Golden Rule - reminder

2nd step: Energy conservation

$$\left(\frac{d^2\sigma}{d\Omega dE'}\right)_{\lambda\to\lambda'} = \frac{k'}{k}\left(\frac{m}{2\pi\hbar}\right)^2 |\langle k'\lambda'|V|k\lambda\rangle|^2 \underline{\delta(E_\lambda - E_{\lambda'} + E - E')}$$

$$\int \delta(E_\lambda - E_{\lambda'} + E - E') = 1$$

3rd step: Integration with respect to neutron coordinate r

$$\langle k'\lambda'|V|k\lambda\rangle = \sum_j V_j(\kappa)\langle\lambda'|e^{i\kappa R_j}|\lambda\rangle \qquad \underline{V_j(\kappa) = \int V_j(x_j)e^{i\kappa x_j}dx_j}$$

$$\kappa = k - k'$$

Interaction = Fourier transform of the potential function

## 3.2 Fermis Golden Rule

4th step: Ansatz: Delta function potential for single nucleus

$$V(r) = a\delta(r)$$

Fermi pseudopotential: $V(r) = \frac{2\pi\hbar^2}{m} b\delta(r)$

- b can be bound or free scattering length!
- Fermi pseudopotential does NOT represent physical reality

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'}\right)_{\lambda\to\lambda'} = \frac{k'}{k}\left|\sum_j b_j \langle k'\lambda' | e^{i\kappa R_j} | k\lambda\rangle\right|^2 \delta(E_\lambda - E_{\lambda'} + E - E')$$

## 3.2 Fermis Golden Rule

5th step: Integral representation of the delta function for energy.
Idea: Stick all the time dependence into the matrix element

$$\delta(E_\lambda - E_{\lambda'} + E - E') = \frac{1}{2\pi\hbar}\int_{-\infty}^{\infty} e^{i(E_\lambda - E_{\lambda'})t/\hbar} e^{-i\omega t} \mathrm{dt}$$

H is the Hamiltonian of the scattering system with Eigenfunctions λ and Eigenvalues $E_\lambda$

$$H|\lambda\rangle = E_\lambda|\lambda\rangle$$

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'}\right)_{\lambda\to\lambda'} = \frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j,j'} b_j b_{j'} \int_{-\infty}^{\infty} \langle\lambda|e^{-i\kappa R_{j'}}|\lambda'\rangle \langle\lambda'|e^{iHt/\hbar} e^{i\kappa R_j} e^{-iHt/\hbar}|\lambda\rangle\, e^{-i\omega t}\mathrm{dt}$$

No terms of λ and λ' outside the matrix element anymore!

## 3.2 Fermis Golden Rule

6th step: Sum over final states, average over initial states

- We use:

$$\sum_{\lambda'} \langle\lambda|A|\lambda'\rangle\langle\lambda'|B|\lambda\rangle = \langle\lambda|AB|\lambda\rangle$$

$$p_\lambda = \frac{1}{Z} e^{\frac{-E_\lambda}{k_b T}}$$

$$\langle A\rangle = \sum_\lambda \langle\lambda|A|\lambda\rangle$$

- Stick the time evolution into the operator for the position $R_j$ $R_j(t) = e^{iHt/\hbar} R_j e^{-iHt/\hbar}$

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega\mathrm{d}E'} = \frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j,j'} b_j b_{j'} \int_{-\infty}^{\infty} \left\langle e^{-i\kappa R_{j'}(0)} e^{-i\kappa R_j(t)} \right\rangle e^{-i\omega t}\mathrm{d}t$$

Correlation function

## 3.2 Fermis Golden Rule: Coherent and incoherent scattering

Assume a large sample with statistically variations of $b_j$

No correlations among $b$: $\overline{b_{j'}b_j} = (\bar{b})^2, j' \neq j$

$$\overline{b_{j'}b_j} = (\overline{b^2}), j' = j$$

Double differential crosssection spilts up into two terms:

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'}\right)_{coherent} = \frac{\sigma_{coh.}}{4\pi}\frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j,j'}\int_{-\infty}^{\infty}\underline{\left\langle e^{-i\kappa R_{j'}(0)}e^{-i\kappa R_j(t)}\right\rangle} e^{-i\omega t}\mathrm{d}t$$

$$\left(\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega \mathrm{d}E'}\right)_{incoherent} = \frac{\sigma_{inc.}}{4\pi}\frac{k'}{k}\frac{1}{2\pi\hbar}\sum_{j,j'}\int_{-\infty}^{\infty}\underline{\left\langle e^{-i\kappa R_j(0)}e^{-i\kappa R_j(t)}\right\rangle} e^{-i\omega t}\mathrm{d}t$$

$$\sigma_{coh.} = 4\pi\bar{b}^2 \qquad \sigma_{inc.} = 4\pi(\overline{b^2} - \bar{b}^2)$$

## 3.2 Fermis Golden Rule: Coherent and incoherent scattering

Coherent

Interference effects:

Bragg scattering

Inoherent

Constant in $Q$

Given by variations in b due to spin, disorder, random atomic motion....