Physics with neutrons 1

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Exercise sheet 1
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Exercise 1.1

The Curie-Joliot hypothesis was introduced in the lecture. It was very controversial from the very beginning. In particular the high proton energy could not be explained: based on Compton-scattering a γ energy of $h\nu = 55\,\mathrm{MeV}$ is needed to produce a 5.7 MeV proton. Such a high γ energy seems to be impossible since the energy of the Po-α radiation is only 5 MeV.
Prove and explain – based on the Compton process – the following expression, which casts the Curie-Joliot hypothesis into doubt:

$$E_p = \frac{2(h\nu)^2}{2h\nu + m_p c^2}$$

The proton rest mass is given by $m_p = 1.6726 \cdot 10^{-27}\,\mathrm{kg}$.

Solution. Photon Energy: $E_\gamma = h\nu = \frac{hc}{\lambda}$

The Compton process is described by

$$\lambda' - \lambda = \frac{h}{m_p c}(1 - \cos\theta).$$

Apparently the scattering angle that transfers the maximum energy to the proton is $\theta = \pi$. We get

$$\Delta\lambda = \frac{2h}{m_p c} \Rightarrow h\nu' = \frac{hc}{\lambda + \Delta\lambda} = \frac{hc}{hc/h\nu + 2h/m_p c} = \frac{h\nu}{1 + 2h\nu/m_p c^2}.$$

With conservation of energy we get for the proton energy

$$E_p = -\Delta E_\gamma = E_\gamma - E'_\gamma = h\nu\left(1 - \frac{1}{1 + 2h\nu/m_p c^2}\right) = \frac{2(h\nu)^2/m_p c^2}{1 + 2h\nu/m_p c^2} = \frac{2(h\nu)^2}{2h\nu + m_p c^2}.$$

□

Exercise 1.2

Complement the following table, assuming nonrelativistic matter waves of neutrons and electrons:

	E [eV]	T [K]	λ [Å]	v [m/s]	$Q_{\max}$ [Å^{-1}]
Light (Red Laser)			6320		
X-rays (Cu K_α)					
Cold neutrons			6		
Thermal neutrons				2200	
Hot neutrons		2300			
Fission neutrons	$2.1\cdot10^6$				
Electrons				$1.57\cdot10^6$	

Fission neutrons are the ones created in the fission. Then the neutrons are converted to other (much lower) energies to be used in scattering experiments.
$Q_{\max}$ is the theoretical maximal momentum transfer that can be reached in an scattering experiment, defined by

$$Q = \frac{4\pi}{\lambda}\sin\left(\frac{2\theta}{2}\right)$$

where 2θ is the scattering angle.

Solution. Table:

	E [eV]	T [K]	λ [Å]	v [m/s]	$Q_{\max}$ [Å^{-1}]
Light (Red Laser)	1.96	22800	6320	299792458	0.00199
X-rays (Cu K_α)	8048	93400000	1.54	299792458	8.16
Cold neutrons	0.00227	26	6	659	2.09
Thermal neutrons	0.0253	294	1.80	2200	6.98
Hot neutrons	0.1981	2300	0.64	6156	13.8
Fission neutrons	$2.1\cdot10^6$	$2.44\cdot10^{10}$	$2\cdot10^{-4}$	$2\cdot10^7$	$6.3\cdot10^4$
Electrons	7.0077	$8.14\cdot10^4$	4.63	$1.57\cdot10^6$	$1.12\cdot10^9$

Apparently, $Q_{\max}$ is reached for $2\theta = \pi$. □

Exercise 1.3

Draw the dispersion relations E [eV] vs. k [Å^{-1}] for light, neutrons and electrons in vacuum, for example with Matlab/Python. Mark the places of all kinds of radiation from Exercise 1.2 (except for the fission neutrons).

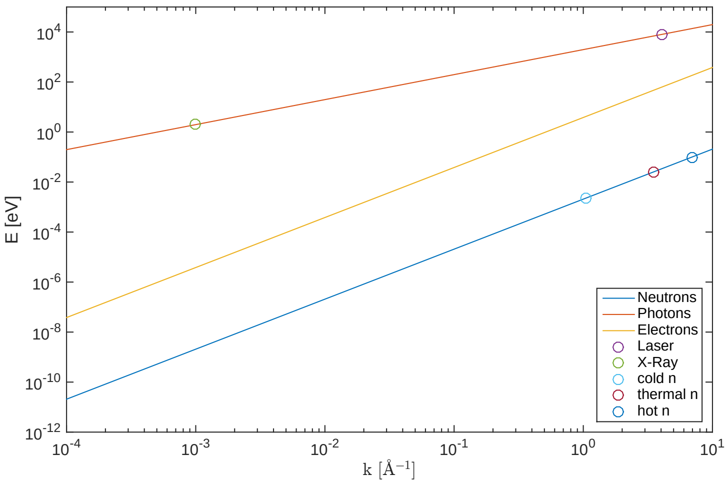
Solution. For matter waves the dispersion relation is given by

$$\hbar\omega = \frac{\hbar^2 k^2}{2m}$$

and for photons

$$\hbar\omega = \hbar v_{\text{ph}} k,$$

with $v_{ph} = c$ in vacuum. Dispersion relations for photons, neutorns and electrons:



□

Exercise 1.4

The δ-function $\delta(f) = \langle\delta|f\rangle = f(0)$ can be defined as the limit of a function series d_l with

$$\delta(f) := \lim_{l\to 0} d_l(f) := \lim_{l\to 0} \langle d_l|f\rangle\,, \qquad \text{(limit after integration)}(*).$$

1. Show that the function series

$$d_l(x) = \begin{cases} 1/l & -l/2 \le x \le l/2 \\ 0 & \text{sonst} \end{cases}$$

   can be used to express the δ-function.

2. Using this definition of the δ-function, calculate its Fourier transform.

Note: The δ-function is not a proper function, due to $(*)$. In mathematics such an object is called distribution.

Solution. We calculate the inner product of an arbitrary continuous, integrable function f with $d_l(x)$ and calculate the limit.

$$\lim_{l\to 0}\int_{-\infty}^{\infty} dx\ d_l(x)f(x) = \lim_{l\to 0}\frac{1}{l}\int_{-l/2}^{l/2} dx\ f(x) = \lim_{l\to 0}\frac{1}{l}l\ f(\bar{x}) = f(0).$$

The integration was performed by means of the mean value theorem, which states the existence of an (unknown) $\bar{x}\in[-l/2, l/2]$ fullfilling the equation. For $l\to 0$ we get $\bar{x}\to 0$.
We calculate the Fourer transform

$$\begin{aligned}\mathcal{F}(\delta)(q) &= \int_{-\infty}^{\infty} dx\ \delta(x)\exp(-ix\cdot q) = \lim_{l\to 0}\int_{-l/2}^{l/2} dx\ \frac{1}{l}\exp\left(-ix\cdot q\right)\\ &= \lim_{l\to 0}\ -\frac{1}{iql}\left[\exp(-ix'\cdot q)\right]_{x'=-l/2}^{l/2} = \lim_{l\to 0}\frac{2i}{iql}\sin(ql/2) = \lim_{l\to 0}\ 2\frac{(ql/2)}{ql} = 1,\end{aligned}$$

i.e.

$$\begin{aligned}\mathcal{F}(\delta) &= \mathbb{1}_{\infty},\\ \delta(x) &= \frac{1}{2\pi}\int_{\mathbb{R}} dk\,\exp(ik\cdot x).\end{aligned}$$

□